Chasing Noise

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Abstract

We present a simple model in which rational but uninformed traders occasionally chase noise as if it were information, thereby amplifying sentiment shocks and moving prices away from fundamental values. In the model, noise traders can have an impact on market equilibrium disproportionate to their size in the market. The model offers a partial explanation for the surprisingly low market price of financial risk in the spring of 2007.

1. Introduction

In the spring of 2007, financial markets, and in particular markets for fixed income securities, were extraordinarily calm. Corporate bond spreads were remarkably low, as were the prices of credit default swaps (CDS) on financial firms (see Fig. 1 below). This tranquility ended in the summer of 2007, as the problems with subprime mortgages precipitated a sequence of events leading to a major financial crisis. The price of risk eventually rose to the highest level in decades.

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It is obvious from the tranquility in the spring of 2007 that financial markets, and in particular, derivative markets, did not anticipate the crisis. What makes this fact particularly interesting is that most of the participants in these markets are sophisticated investors. Unlike, say, in the Internet bubble, this pricing was unlikely to be driven by the mass of demand by unsophisticated investors. Could the observed tranquility of markets in the spring of 2007 have resulted from the trading behavior of sophisticated investors that masked the potential bad news? In this paper, we suggest that the answer is yes. We propose a very simple model, extending Grossman and Stiglitz (1980), which focuses on the interaction of different types of investors in a market, the vast majority of whom are rational, and shows how this interaction can sustain incorrect prices.

The basic idea is to consider three types of investors: a small number of investors, called Insiders, who possess valuable information and trade completely rationally, a small number of Noise traders who are vulnerable to sentiment shocks and trade on those, and the vast majority of Outsiders, who possess no information but learn from prices and trade rationally. All the Insiders have the same information, and all the Noise traders face the same sentiment shock. The focus of the paper is the trading by the silent majority of Outsiders, and its effect on prices.

The problem facing an Outsider is difficult. On the one hand, he wants to follow the Insiders who know something, but since he only observes prices, would like to chase price increases caused by Insiders trading on valuable information. On the other hand, he wants to bet against the Noise traders who are influenced by sentiment, but again since he only observes prices, would like to sell into a rising market and be a contrarian. Which one of these motives dominates? In particular, is it possible for this rational Outsider to get confused and to chase noise as if it were information? We show that in markets with sufficiently few Noise traders, the answer is yes, and Outsiders occasionally end up chasing sentiment, thereby suppressing the possible impact of informed trading on prices. They do so because, in those circumstances, they believe that price movements reflect information even though they reflect noise.

The composition of a market can be depicted graphically on a triangle, as in Fig. 2. The x-axis represents the proportion of market participants who are Insiders, denoted by I, and the y-axis represents the proportion who are Noise traders, denoted by N. The remaining proportion is Outsiders, denoted by O, so that the three shares sum to one. Both Insiders and Outsiders are

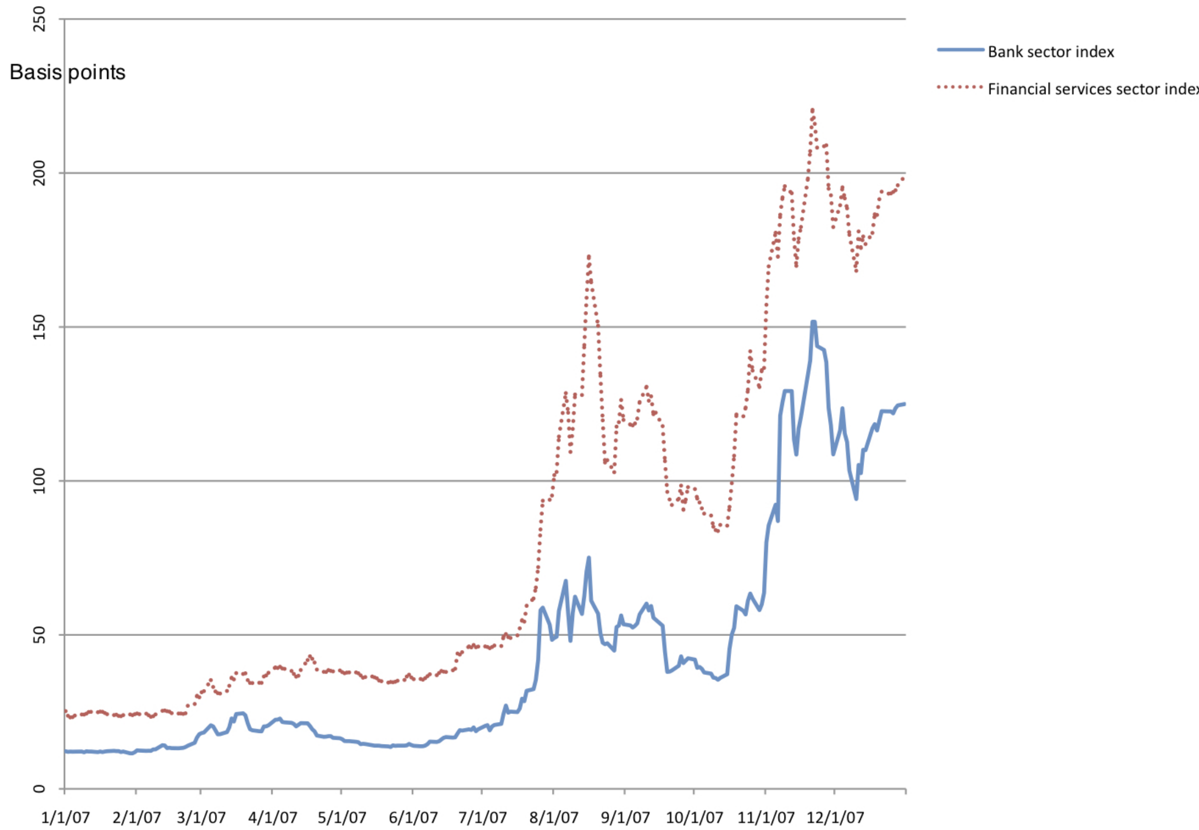

Fig. 1: Credit default swap (CDS) spreads 1/1/07-12/31/07. These are constructed as a weighted average of CDS spreads for individual firms in the Banking and Financial services sectors.

sophisticated, but the former are better informed. In the markets of interest, we think of most traders as Outsiders: rational and sophisticated, but not well-informed. This corresponds to points near the origin in this triangle, labeled "Region of interest."

We can think of the evidence in Fig. 1 as an outcome in a market in our Region of interest. Specifically, the corporate bond and CDS markets are dominated by Outsiders, with small but positive masses of Noise traders and informed traders. In the spring of 2007, the Noise traders were very calm (and hence very willing to sell insurance), and the majority of sophisticated but uninformed investors took the low price of risk as evidence that the world

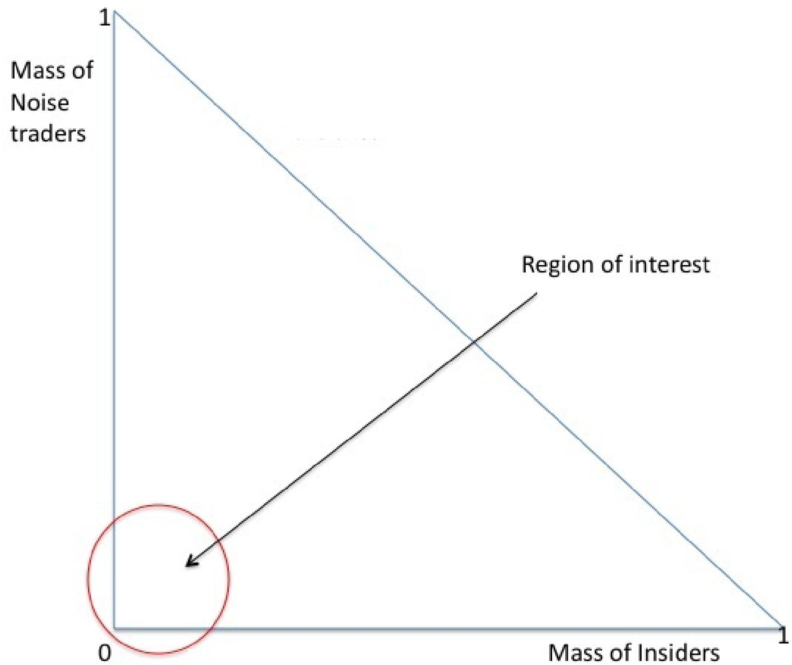

Fig. 2: Market composition. The masses of Insiders, Noise traders, and Outsiders sum to one. The Region of interest is the area near the origin but off the axes.

was indeed safe. As a consequence, they were also willing to sell insurance. Even if there were informed investors in this market who saw the risk of a calamity and were buying insurance, their demand was constrained by their risk-bearing capacity (Lewis, 2010). This demand was then insufficient to raise the price of risk significantly because the Outsiders owned most capital and believed that the low price of risk reflected good news. Starting in the summer of 2007, public news about fundamentals revealed that the low price of risk was not justified. Because risk was not correctly priced before, the reaction of the price to news was substantial, as Fig. 1 shows.

We examine the responsiveness of prices to sentiment when almost all investors are sophisticated. If S is the sentiment shock and p is the price of the asset, our argument requires that $\frac{\partial p}{\partial S}$ be large. One might think that this will not be true in a market with only a few Noise traders, because such a market will behave almost like a market with no Noise traders at all. We show how this intuition can fail. Under plausible conditions, $\frac{\partial p}{\partial S}$ can be very large in our Region of interest. This implies that the small mass of Noise traders can have a disproportionately large impact on market prices. Even with a modest Noise trader shock, prices can diverge sharply from fundamental values in a market dominated by sophisticated traders.

This counterintuitive result holds because the Outsiders, in their attempt to chase the Insiders, occasionally chase Noise traders instead. In markets with a high enough average information-to-noise ratio, each Outsider's demand curve is upward sloping. Since there is a large mass of these traders, they exert strong pressure on prices in the direction where they observe movement.

Without Noise traders, the fully revealing equilibrium of Grossman and Stiglitz (1976) prevails. Outsiders observe the price and infer the predictable component of the fundamental value: they fully trust the price. With the first marginal mass of Noise traders, prices still reflect mainly information and Outsiders still heavily rely on the price in their expectation formation. Their expectations still move almost one-for-one with movements in prices and thus almost one-for-one with noise. They trade on this information and hence amplify price movements due to noise.[1]

We consider three metrics for stability[2] and efficiency of the market. The first is an ex-post measure of the responsiveness of price to the Noise trader shock, $\frac{\partial p}{\partial S}$. The second is an ex-ante measure of the variance of the price, conditional on the Insider's information, $Var(p|InsideInformation)$. The third is the informativeness of the pricing system, as defined by Grossman and Stiglitz, $corr(value, p)$. According to this last metric, additional Insiders make the market more efficient, on average. However, we are especially interested in the first two metrics, because they speak to the question of how markets can deviate from efficiency even when most traders are sophisticated and Noise trader shocks are modest. We consider these metrics separately to distinguish between ex-ante and ex-post stability.

If we see a single outcome in which the market price is apparently far away from fundamental value–like credit default swaps in the spring of 2007–and

[1] We thank an anonymous referee for suggesting this exposition.

[2] A stable market is not very responsive to noise and has low levels of noise-induced variance.

want to understand how and why this happened, it is helpful to know that $\frac{dp}{dS}$ is very large so a moderate sentiment shock could plausibly cause a significant mispricing. We study $\frac{dp}{dS}$ to understand the magnitude of mispricing in individual realizations of the model.

We are also interested in the long-run properties of market behavior. The model is static, but we can think intuitively about market behavior over time as many repeated outcomes from the model. For this, the important metrics are moments of the data: the informativeness of the pricing system, $Var(p)$, or $Var(p|InsideInformation)$. High informativeness or low conditional variance are indicative of a market that behaves well, on average, over time.

The literature on trading in financial markets between better-and less-informed investors is huge, so we can only refer to some of the studies. Grossman and Stiglitz (1980) consider a model with only rational investors and demonstrate that, when acquiring information is costly, there cannot be a market equilibrium in which prices fully reflect fundamental values. Because we are interested in a different question than Grossman and Stiglitz, we do not consider the aggregation of information from differentially informed rational traders. Rather, we focus on the efforts of uninformed rational traders to piggyback on the trading of the informed ones.

Kyle (1985) considers markets with informed investors and Noise traders, but also an uninformed but rational investor who, in his case, is a market maker. Kyle is interested in market microstructure, and hence focuses on the behavior of a monopolistic risk-neutral market maker, a setting appropriate for his objective. We in contrast are interested in the market interactions of small competitive investors, and hence have a different model and different results. Wang (1993) presents a dynamic trading model with differentially informed investors, and shows that less-informed investors can rationally behave like price chasers. His model incorporates effects similar to ours, but does not focus on the extreme sensitivity of prices to noise in the Region of interest. Kogan, Ross, Wang, and Westerfield (2006) examine the connection between Noise traders' survival and their impact on market equilibrium. They find that the two are not as tightly linked as naive intuition would suggest. As Noise trader wealth goes to zero over time, their price impact can decline much more slowly. Their results are similar to ours in that they find Noise trader impact can be disproportionate to their wealth, although their mechanism focuses on the type of trading irrational traders engage in, rather than interaction effects. Barlevy and Veronesi (2003) consider a model with risk-neutral Outsiders trading with Noise traders and Insiders, optimally extracting information from the price of an asset. In their model, the Outsiders have a non-monotonic demand curve, leading the relationship between price and fundamentals to be S-shaped. This induces a discontinuity in price when the fundamentals fall below a certain level, which Barlevy and Veronesi interpret as a crash. Their mechanism is different from ours, but their market structure is similar.

In Stein (1987), rational speculation can impose an externality on traders trying to make inferences from prices, and consequently destabilize prices. In Calvo (2002), rational uninformed investors optimally extract information from prices affected by informed investors. Instead of being confounded by the presence of Noise traders, the confound he considers is occasional liquidity shocks to the informed traders forcing them to withdraw from the market. The uninformed traders misinterpret this as a negative shock to fundamentals and drive down prices.

Our paper is also related to the literature on noise trading. DeLong et al. (1990a) model the interaction between rational speculators, who would correspond to the Outsiders in our model, and Noise traders. With no Insiders in that model, trading by speculators unambiguously stabilizes prices. In DeLong et al. (1990b), arbitrageurs buy in anticipation of positive feedback trading by the Noise traders, and thus destabilize prices.[3] Allen and Gale (1992) present a model of stock price manipulation by a large investor, who buys and thus stimulates demand by uninformed investors trying to infer information from price movements. Rossi and Tinn (2010) use the Kyle (1985) framework to model positive feedback trading by rational uninformed investors trying to learn from prices. Their model has several periods and a different setup than ours, but they are trying to get at some related ideas on how uninformed but rational speculators balance their desires to follow

[3]We are assuming that all the traders are price-takers, but in the limiting case when we are thinking of literal Insiders, it is worth thinking about the possibility of price-manipulation. In the two-period model we consider, the Insiders problem turns into roughly that faced by Kyle's (1985) Insider, so he trades less aggressively to make monopoly profits. To get more intricate price-manipulation behavior, we would need a longer-horizon dynamic model as in DeLong et al. (1990b). It is difficult to imagine equilibria in which Insiders systematically manipulate prices to take advantage of the Outsiders upward-sloping demand curve. In such a strategy profile, it will generally be optimal for the Outsider to deviate to submit a downward-sloping demand curve.

Insiders and to bet against Noise traders.

Stein (2009) considers arbitrageurs trading against a statistical regularity (under-reaction) causing a new type of market inefficiency in the process of trading away profit opportunities on the old type. He shows that prices can sometimes be further away from fundamental values than they are without the arbitrageurs. In both his approach and ours, rational traders try to push prices towards their rational expectation of fundamental value, but in our approach the expectation of fundamental value derives from both a private signal and observation of the price, whereas his traders observe the price and a statistical regularity they can take advantage of.

The rest of the paper proceeds as follows. In Section 2 we formally present and solve the model. Section 3 examines the slope of an Outsider's demand curve. Section 4 analyzes the implications of the demand curve for market equilibrium. Section 5 considers measures of market stability and efficiency besides the sensitivity of market price to sentiment. Section 6 concludes. All proofs and derivations are in the appendices.

2. The model

There is a market for a risky asset in supply 1 trading at price p. There are two periods. Trading occurs in period 1, then the asset pays off its fundamental value V in period 2.[4] The fundamental value is the sum of three terms. First is the unconditional expectation μ.[5] Second is a shock

[4]We think of period 1 as a length of time over which all the agents trade anonymously and repeatedly until the market settles into equilibrium. In any such situation, Insiders and Noise traders will make initial trades. If their demand curves are upward sloping, the Outsiders will trade in the direction of the subsequent price movements, and the sequential behavior of trades may resemble that in a model of rational herding (see Bikhchandani et al., 1992; Froot et al., 1992). We do not model these dynamic interactions.

[5]As Malcolm Baker pointed out, we could imagine a situation in which Outsiders have more variables in their information set (such as ratings on structured finance), just fewer than the Insiders. We could then write the value V as $V = \mu + \nu_0 + \sigma_1\nu_1 + \sigma_2\nu_2$, where ν_0 is observable by all the agents. But this is equivalent to a renormalization of the constant μ to $\mu+\nu_0$. Up to redefinition, any other variables we include in the model which are common knowledge become part of μ.

If instead, the signal ν_0 is observable only to the Insiders and the Outsiders, the equivalence is more complicated because we need to add a conditional mean of $-\nu_0$ to the Noise trader shock S. The results are essentially unchanged. We are therefore sweeping under the rug the majority of the information available to the traders by putting it into

$\sigma_1\nu_1$ which is realized in period 1. ν_1 is Normally distributed with mean zero and variance 1. Finally, there is a shock $\sigma_2\nu_2$ to fundamental value which is not realized until the second period. ν_2 is also distributed Normally with mean zero and variance 1. The fundamental value is then given by

$$V = \mu + \sigma_1\nu_1 + \sigma_2\nu_2. \tag{1}$$

In addition to this risky asset, there is a riskless asset in elastic supply with return r.

There are three types of agents participating in this market: a mass N of Noise traders, I of Insider/informed traders, and O of Outsiders/uninformed sophisticated traders. We normalize $I + O + N = 1$. In period 1, the Insider traders get a signal about the termination value of the asset. That is, each Insider observes the same ν_1.

The Noise traders do not learn from prices and have a biased belief about the fundamental value of the asset, given by a shock to their level of "sentiment," the random variable S.[6] S is distributed normally with mean zero and variance σ_S^2. Every Noise trader has the same realization S. S is independent of all fundamentals.[7]

Outsiders are rational and optimally interpret the price signals they observe. All agents have constant absolute risk aversion utility with parameter γ.

We begin by deriving the period-1 demand curves directly from utility maximization. Each agent i begins with wealth W_i and chooses demand D_i to maximize

$$E_i[-e^{-\gamma(D_iV+(W_i-D_ip)r)}].$$

the constant μ. Any information revelation or fundamental research that occurred before period 1 is important economically, but does not bear on the interactions we consider.

[6] An alternative approach to modeling the idea that sophisticated traders react to noise is dispersed information. In those models, strategic complementarities cause traders to partially coordinate based on a noisy public signal such as a price, so noise in the public signal can be substantially magnified as each trader reacts to the others' actions. See Allen, Morris, and Shin (2006), Angeletos and Pavan (2007), Angeletos and La'o (2009), and Hassan and Mertens (2010), among others. In particular, Mertens (2008) finds that with dispersed information, small distortions in beliefs can render arbitrage infeasible.

[7] In the CDS market in 2007, we would argue that the price of risk diverged from fundamentals, caused by a misperception about the riskiness of the underlying, which implies a misperception about the expected return on the derivative.

Maximizing this expression is equivalent to minimizing minus this expression, which is in turn equivalent to minimizing the log of that. Assuming for the moment that V is normally distributed *conditional on agent i's information set*, the first-order condition immediately gives the demand curve:

$$D_i = \frac{E_i[V] - pr}{\gamma \sigma_i^2(V)}, \tag{2}$$

where $E_i[\cdot]$ denotes the expectation with respect to agent i's information set and $\sigma_i^2(V)$ denotes the variance of V conditional on agent i's information set. For the Insider, this becomes

$$D_I = \frac{\mu + \sigma_1 \nu_1 - pr}{\gamma \sigma_2^2}. \tag{3}$$

For the Outsider, this becomes

$$D_O = \frac{\mu + E[\sigma_1 \nu_1 | p] - pr}{\gamma \sigma_O^2}, \tag{4}$$

where $E[\sigma_1 \nu_1 | p]$ and σ_O^2 are endogenous. σ_O^2 is given by

$$\sigma_O^2 = Var(\sigma_1 \nu_1 | p) + \sigma_2^2. \tag{5}$$

Finally, the demand for the Noise traders is given by

$$D_N = \frac{\mu + S - pr}{\gamma \sigma_N^2}, \tag{6}$$

where σ_N^2 is the variance perceived by the Noise traders. Since the Noise traders do not observe a signal or use the price to update their information set, their perceived variance is the same[8] as the ex-ante variance $\sigma_N^2 = (\sigma_1^2 +$

[8]In the model, we give each trader the same risk aversion and exogenously specify the Noise trader's perceived variance as $\sigma_1^2 + \sigma_2^2$. We could imagine situations in which the level of risk aversion varied across types or in which the Noise trader's incorrect beliefs extended beyond the first moment of the asset's value. Making these transformations turns out to be equivalent to further altering the composition of the market.

Consider replacing the mass of the Outsiders (or Noise traders or Insiders, respectively) with their mass divided by their risk-aversion parameter and call these C_N, C_O, C_I. Let C be the sum of these terms. For all purposes we consider, this market is equivalent to one with homogeneous risk aversions equal to one and $\frac{1}{C}$ units of the asset. Since the supply of the asset affects only the equilibrium risk premium, which we do not consider, this is equivalent to varying the composition of the market, putting more weight on the

σ_2^2). With all this in hand, we can proceed to solve the model. Imposing market clearing and rearranging gives

$$\gamma-\mu(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})+pr(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})-\frac{O}{\sigma_O^2}E[\sigma_1\nu_1|p]=\frac{N}{(\sigma_1^2+\sigma_2^2)}S+\frac{I}{\sigma_2^2}\sigma_1\nu_1. \tag{7}$$

We can solve the signal extraction problem to find the expectation[9] of $\sigma_1\nu_1$ given p. It is given by

$$E[\sigma_1\nu_1|p]=\frac{\sigma_2^2}{I}\frac{(\frac{I}{\sigma_2^2}\sigma_1)^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2}\times signal, \tag{8}$$

where the signal is proportional to the difference between the left-hand side of (7) and its unconditional expectation. A complete derivation is given in Appendix A. In equilibrium, the conditional expectation and variance are given by

$$E[\sigma_1\nu_1|p]=\frac{\frac{I}{\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}(pr(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})-\mu(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})+\gamma), \tag{9}$$

$$\sigma_O^2=\sigma_1^2\frac{N^2\sigma_S^2\sigma_2^4}{I^2\sigma_1^2(\sigma_1^2+\sigma_2^2)^2+N^2\sigma_S^2\sigma_2^4}+\sigma_2^2. \tag{10}$$

Plugging this back in to the market clearing equation and solving for the price gives

$$p=r^{-1}(\mu-A^{-1})+\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}S+\frac{I}{ABr\gamma\sigma_2^2}\sigma_1\nu_1, \tag{11}$$

where we have defined A and B as

$$A=\frac{O}{\gamma\sigma_O^2}+\frac{I}{\gamma\sigma_2^2}+\frac{N}{\gamma(\sigma_1^2+\sigma_2^2)}, \tag{12}$$

less risk-averse participants. Changing the variance perceived by the Noise traders works identically, because this variance enters their demand only multiplicatively with their risk aversion.

[9] We show in Appendix A that there are situations in which the Outsiders may have a higher expectation of fundamental value than *either* the Insiders or the Noise traders. This is another sense in which markets can be considered unstable.

$$B = 1 - \frac{\frac{OI}{\sigma_O^2 \sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2 + \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}. \tag{13}$$

In (11), r^{-1} appears in each term because it is the riskless discount factor. A is a factor describing the aggregate risk-bearing capacity of the market, the inverse of which corresponds to the risk-premium agents demand in equilibrium in the first term.

The second term is the impact of the Noise trader sentiment shock on the market price. The coefficient here is $\frac{\partial p}{\partial S}$ and will be the subject of some examination. The third is the impact of the aggregate information about fundamental value on the price. If the market resembles the Noise trader-free benchmark, the coefficient on S should be close to zero and the coefficient on ν_1 should be close to $\sigma_1 r^{-1}$.

The ultimate objects of interest are how completely the fundamental information ν_1 and the sentiment S are incorporated into the prices of the asset. We can write the impact of the fundamental information ν_1 and sentiment shock S as

$$\frac{\partial p}{\partial \nu_1} = \frac{I\sigma_1}{ABr\gamma\sigma_2^2}, \tag{14}$$

$$\frac{\partial p}{\partial S} = \frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}. \tag{15}$$

It is difficult to evaluate these expressions analytically. In thoroughly studied special cases, there are either only Insiders and Outsiders or only Noise traders and Outsiders. In the former case, the coefficient on ν_1 does turn out to be $\sigma_1 r^{-1}$, while in the latter case the coefficient on S decreases towards zero as the risk-bearing capacity of the sophisticated traders increases. These are signs of a stable market that prices assets effectively.

From these observations, the natural intuition to build would be that adding more sophisticated investors, and in particular, adding more informed sophisticated investors, pushes the coefficient on S towards zero and decreases the market volatility. Similarly, intuition might suggest that a small N necessarily implies a small coefficient on S, so Noise trader shocks do not get factored into the price of the asset.

As we show in Section 5, neither of these intuitions holds for markets in the Region of interest. The reason for this is that prices in this model are driven primarily by the trading behavior of the Outsiders, who have most of the risk-bearing capacity and hence ability to move prices in this model. Outsiders are trying to chase information, but may occasionally end up chasing noise. Their efforts to chase information make them more aggressive when they think there is more information in the market, which means that adding Insiders to the market might destabilize prices. These efforts to chase information also lead them to chase noise in some circumstances by mistake, which might also have a destabilizing influence. In the analysis below, we seek to develop this logic.

To this end, we focus on evaluating $\frac{\partial p}{\partial S}$ in the Region of interest. In Appendix B, we prove the following lemma:

Lemma 1.

$$\frac{\partial p}{\partial S} = (\frac{I}{\sigma_2^2}+\frac{N}{\sigma_1^2+\sigma_2^2})^{-1}\frac{N}{r(\sigma_1^2+\sigma_2^2)}+(\frac{I}{\sigma_2^2}+\frac{N}{\sigma_1^2+\sigma_2^2})^{-1}r^{-1}\gamma O\times OutsiderDemandCurveSlope, \tag{16}$$

where the $OutsiderDemandCurveSlope$ is defined as $\frac{\partial D_O}{\partial p}$. Lemma 1 makes it clear that the slope of an Outsider's demand curve is crucial for stability of financial markets, as proxied for by $\frac{\partial p}{\partial S}$. Our first step, then, is to examine this slope.

3. The slope of the outsider demand curve

In the cases of interest, Outsiders compose most of the market. As suggested by Lemma 1, their demand curve and its slope in particular are then important to understanding to see how the market behaves. The slope of an Outsider's demand curve (after some rearrangement) is given by

$$\frac{dD_O}{dp} = \frac{r}{\gamma}\frac{1}{\frac{\sigma_1^2}{\sigma_2^2}I(1-N)+\frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)}}\frac{N}{(\sigma_1^2+\sigma_2^2)}(\frac{I}{\sigma_2^2}\sigma_1^2-\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S^2). \tag{17}$$

We can understand the demand curve better by looking at its three multiplicands separately. The third term is the easiest to interpret, as it determines the sign of the slope. Specifically,

$$(\frac{rI}{\gamma\sigma_2^2}\sigma_1^2-\frac{rN}{\gamma(\sigma_1^2+\sigma_2^2)}\sigma_S^2),$$

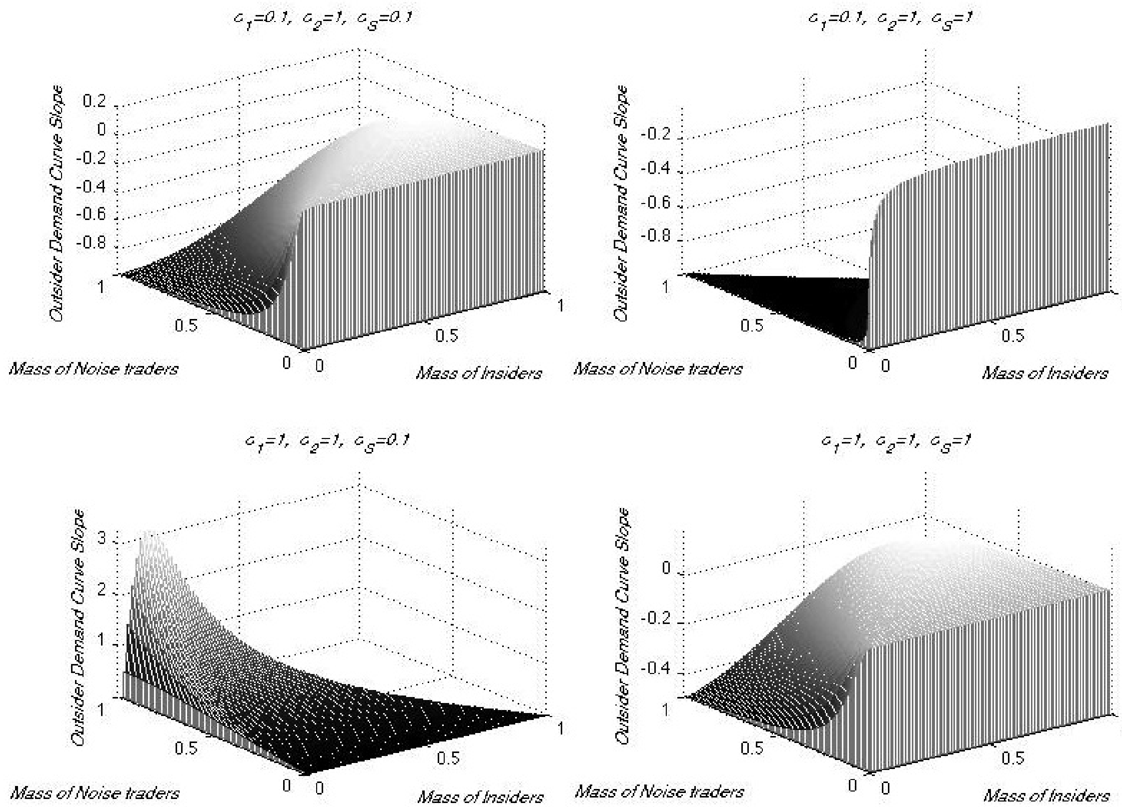

Fig. 3: Outsider demand curve slope. The slope of the Outsider's demand curve depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

is the slope of the aggregate Insider demand curve times the variance of their signal minus the slope of the aggregate Noise trader demand curve times the variance of their "signal." If the Noise traders are "noisier" than the Insiders are "inside," then the demand curve will be downward sloping.

The middle term

$$\frac{rN}{\gamma(\sigma_1^2 + \sigma_2^2)},$$

is the slope of the aggregate Noise trader demand curve. When this slope is small, the Noise trader demand is highly inelastic, so it is difficult to trade with them without changing the price significantly. This makes it harder to trade against Noise trader irrationality, a significant source of equilibrium profits for the Outsiders. Limited ability to make profits from the Noise traders dampens the Outsider's willingness to trade, making his demand curve less steep.

When this slope is large, the Outsiders can gain a lot by trading against Noise traders. When this slope is small, the Noise traders make it hard to trade against them so the Outsider's demand curve is less steep.

The first term is harder to interpret:

$$\frac{1}{\frac{r\sigma_1^2}{\gamma\sigma_2^2}I(1-N)+\frac{rN^2\sigma_S^2}{\gamma(\sigma_1^2+\sigma_2^2)}}.$$

The second term in the denominator is N times the slope of the aggregate Noise trader demand curve times the variance of their shock. The first term is the slope of the aggregate Insider demand curve times the variance of their shock, times $I(1-N)$, which is a term describing the interaction between the Insiders and the Outsiders trying to emulate them.

We would like to understand this demand curve in terms of three effects: the Outsiders trying to trade against the Noise traders, trying to avoid adverse selection from better-informed Insiders, and trying to trade with Insiders when they have a strong signal.

We interpret the middle term as being solely a matter of trading against Noise traders. This makes sense, as this term does not involve the Insiders and so cannot have anything to do with them.

The expression $\frac{I\sigma_1^2}{\sigma_2^2}$ appears in the Outsider's demand curve both additively and multiplicatively. In the third term, it describes the portion of information due to Insiders, which increases the Outsider's desire to trade with Insiders, driving up the slope. We therefore interpret this term as a following-Insiders or positive-feedback effect.

The expression also appears in the denominator of the first term, and it is large when Insiders are aggressive traders. The effect of a big term here is to make the slope flatter, regardless of its sign. When there is enough information in the market for the curve to be upward sloping, a large $\frac{I\sigma_1^2}{\sigma_2^2}$ makes it *less* upward sloping. When there is not enough information in the market for the Outsiders to have an upward-sloping demand curve, this term makes their demand curve *less* downward sloping. We interpret this term as the adverse selection term. Whenever Insiders are aggressive, it makes Outsiders less aggressive because they are afraid of trading against Insiders.

We can directly evaluate the Outsider demand curve slope at $N = 0$ and $I = 0$ to see how the Outsiders behave in the simple cases:

$$I = 0 \Rightarrow \frac{dD_O}{dp} = -\frac{r}{\gamma(\sigma_1^2 + \sigma_2^2)}, \tag{18}$$

$$N = 0 \Rightarrow \frac{dD_O}{dp} = 0. \tag{19}$$

These are reassuring. With only Noise traders to trade against, the Outsider's demand is very elastic, since they know that trading against Noise traders is optimal because prices contain no new information. With only Insiders to trade with, the demand curve is perfectly inelastic because prices are fully revealing and everyone behaves like an Insider (no-trade theorem intuition applies). The separating case is easy to identify:

Lemma 2. *The slope of the Outsider's demand curve is positive if and only if* $\frac{I}{\sigma_2^2}\sigma_1^2 > \frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S^2 > 0$.

Lemma 2 says the Outsider's demand curve is upward sloping if the expectation of the proportion of a price move due to Insiders is greater than the proportion due to Noise traders. In particular, for every market with a nonzero number of Insiders, there is a $\underline{N} > 0$ such that the Outsider's demand curve is positively sloping whenever $0 < N < \underline{N}$. Moreover, it can be shown that for a fixed positive number of Noise traders, more Insiders always means a higher slope of the demand curve.

In the next section we consider the implications of this Outsider behavior on market equilibrium.

4. Market equilibrium

We are interested in whether it is possible for $\frac{\partial p}{\partial S}$ to be large in the Region of interest. We know that it is generally very small on the axes because the sophisticated traders effectively trade against the Noise traders. The general expression for $\frac{\partial p}{\partial S}$ is difficult to analyze in the interior of the domain, so we take three alternative approaches.

First, we analyze the special cases that we do understand well: markets with either no Noise traders or no Insiders. By understanding these markets thoroughly, we can gain insights into the behavior of markets with similar compositions.

Second, we perform local experiments: we ask how $\frac{\partial p}{\partial S}$ changes as we move infinitesimally away from one of our well-understood cases. The market with no Insiders has a very small $\frac{\partial p}{\partial S}$, as does the market with no Noise traders. We ask how $\frac{\partial p}{\partial S}$ changes when we add the marginal Insider or Noise trader. These two experiments are depicted in Fig. 4.

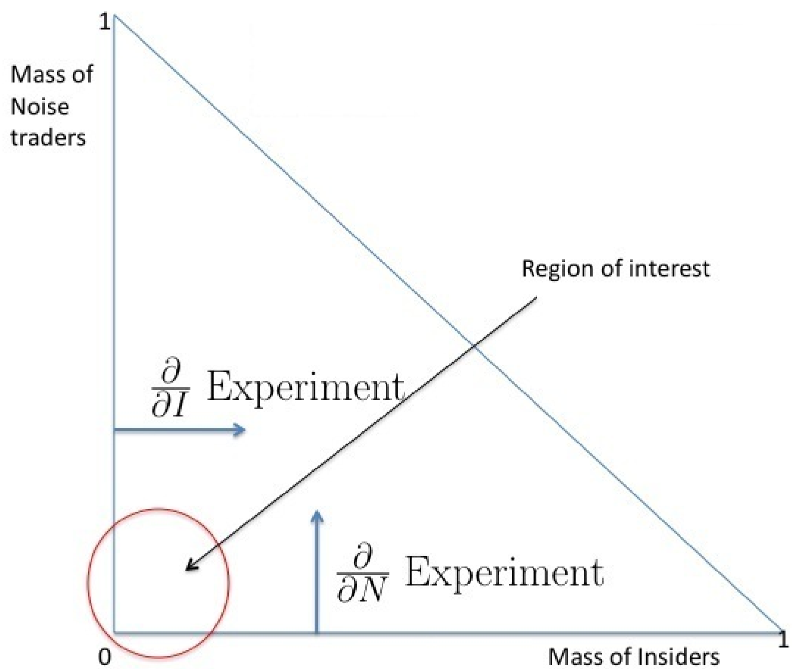

Fig. 4: Changing the market composition. We consider what happens locally as we move from markets with no Noise traders (Insiders) to markets with very few Noise traders (Insiders).

The final approach is numerical. We calculate $\frac{\partial p}{\partial S}$ for a range of parameter values and across the Region of interest to establish that $\frac{\partial p}{\partial S}$ can in fact achieve a maximum near the origin.

4.1. The cases of the missing types

To gain more insight into the market equilibrium, we evaluate the comparative statics of price in the cases in which either Noise traders, Insider traders, or uninformed sophisticated traders are missing. First, suppose Noise traders are absent. When $N = 0$, note that $\sigma_O^2 = \sigma_2^2$, so the expressions for the impact of information and sentiment on price become

$$\frac{\partial p}{\partial \nu_1} = \frac{\sigma_1}{r}, \tag{20}$$

$$\frac{\partial p}{\partial S} = 0. \tag{21}$$

This is intuitive. With no noise coming from the Noise traders, the uninformed investors can perfectly back out the signal ν_1, so they behave as if they are informed. Now, setting $I = 0$ to get rid of the Insider traders and noting that this implies $\sigma_O^2 = \sigma_1^2 + \sigma_2^2$, the comparative statics become

$$\frac{\partial p}{\partial \nu_1} = 0, \tag{22}$$

$$\frac{\partial p}{\partial S} = \frac{N}{r}. \tag{23}$$

Again this is an intuitively appealing result. The Outsiders know that any price movement is due to Noise traders so choose to trade against it, but their ability to do so is limited by their risk-bearing capacity. Their collective risk-bearing capacity depends on their mass O, which is pinned down here to be $1 - N$. Thus, the $O + N$ term disappears from the denominator.

Finally, we can look at the situation with only Insiders and Noise traders, so $O = 0$:

$$\frac{\partial p}{\partial \nu_1} = \frac{I\sigma_1}{r\sigma_2^2}\left(\frac{I}{\sigma_2^2} + \frac{N}{(\sigma_1^2 + \sigma_2^2)}\right)^{-1}, \tag{24}$$

$$\frac{\partial p}{\partial S} = \frac{N}{r(\sigma_1^2 + \sigma_2^2)}\left(\frac{I}{\sigma_2^2} + \frac{N}{(\sigma_1^2 + \sigma_2^2)}\right)^{-1}. \tag{25}$$

The intuition for these results is exactly as above. These results make clear that the model we present subsumes as a special case the previously studied models. Each of the three possible pairings has been studied separately, and we are looking at what happens when all three types are present.

4.2. The first noise trader

When there are no Noise traders in the market, we know $\frac{\partial p}{\partial S}$ is zero. The main contention of this paper is that markets with very small numbers of Noise traders need not behave qualitatively like markets with none at all.

To quantify this claim, we can look at the difference in $\frac{\partial p}{\partial S}$ when we go from $N = 0$ to $N > 0$. To keep the size of the market constant, we perform this experiment holding the number of Insiders constant and changing an Outsider into a Noise trader. That is, $dO = -dN$. This is a comparison of the equilibrium behavior of two different but similarly composed markets. The strongest possible proof of our claim would be a discontinuous jump. This does not occur, but the next strongest proof would be a very high derivative at 0. In Appendix B we prove that this is exactly what we see:

Proposition 1. $\frac{\partial^2 p}{\partial S \partial N}|_{N=0} = \frac{\sigma_2^2}{Ir(\sigma_1^2+\sigma_2^2)}$. *In particular,* $\frac{\partial^2 p}{\partial S \partial N}|_{N=0}$ *becomes arbitrarily large for small* I*.*

Since we generally think of the Insiders as being a small population, this proposition focuses on the most relevant part of the domain. In this region, the first marginal Noise trader can have an enormous impact on market equilibrium despite being infinitesimally small himself.

The driving force behind this result is the positive slope of the Outsider's demand curve.[10] At $N = 0$ the Outsider's demand curve is flat. By Lemma 2, adding a sufficiently small number of Noise traders will make the Outsider's demand curve strictly upward sloping. With an upward-sloping demand curve, the Outsiders will trade with any price movement they observe. When the Noise trader does start trading, the Outsiders chase him trading very aggressively, mistaking it for an Insider trade. This causes the sentiment S

[10]In general, changing the composition of the market will also affect the Outsider's demand curve slope by changing σ_O^2 in the denominator. In Appendix B we show that this is irrelevant for this particular experiment because $\frac{\partial \sigma_O^2}{\partial N}|_{N=0} = \frac{\partial \sigma_O^2}{\partial I}|_{I=0} = 0$. Starting from the boundary, the first Noise trader or Insider has no first-order effect on the variance perceived by the Outsider.

to be factored into the price much more strongly than it would if only the Noise trader were trading on it.

Subsequent Noise traders do not have nearly as big an effect because the Outsider's demand curve flattens and eventually becomes downward sloping as more and more Noise traders join the market. Nevertheless, this proposition captures the fact that it does not take many Noise traders to get a noisy market.

This big effect only comes into play because the Outsider's demand curve is upward sloping at $N \approx 0$. This highlights the centrality of the presence of Insiders. Without them, this slope would not be positive and the effects of noise would not be nearly so pronounced. This suggests that there may be circumstances in which adding Insiders can destabilize the market. We show exactly that in the next section.

4.3. Destabilizing Insiders

In a market with only Noise traders and Outsiders, the Outsiders know any price movement to be caused by the Noise traders, so they trade against any price movements they observe. Their demand curves are strongly downward sloping. Outsiders' willingness to keep the Noise traders from affecting market prices is limited only by their risk-bearing capacity. What happens when we start adding Insiders? In a perfect world, two nice things would happen. First, the Insiders' information would be factored into the price perfectly. Second, the Insiders, who have a lower perceived variance and thus a higher risk-bearing capacity, would effectively trade against any Noise trader shocks.

To examine this, we look at how $\frac{\partial p}{\partial S}$ changes when we add dI Insiders. Holding the number of Noise traders constant so that $dI = -dO$, the experiment we are considering is turning an Outsider into an Insider.

The derivative is hard to evaluate in general analytically, but can be signed locally near $I = 0$ because of the fact that $\frac{\partial \sigma_O^2}{\partial I}|_{I=0} = 0$. In Appendix B we prove the following proposition

Proposition 2. *For sufficiently small levels of σ_S^2, N, and I, increasing the number of Insiders while decreasing the number of Outsiders increases price instability, i.e., $\frac{\partial^2 p}{\partial S \partial I} > 0$.*

Instead of decreasing the impact of Noise traders, adding an Insider increases it. This effect holds in particular in the Region of interest near the origin, where there are many Outsiders. The intuition for this result is twofold. First, when Insiders join the market, the Informativeness of prices to the Outsiders goes up quickly. In particular, if σ_1 is large compared to σ_S and the Insiders are more inside than the Noise traders are noisy, the slope of the Outsider's demand curve quickly shifts upward. The first marginal Insider is not enough to make the demand curve slope up, but as the curve shifts towards flatness, the Outsiders stop trading against the Noise traders, so the Noise traders have a greater impact. This effect is magnified by the fact that in the Region of interest there are many Outsiders all trading with the same strategy.

Second, after enough Insiders have been added to the market, the Outsider demand curve becomes upward sloping. Once this happens, they start actively trading with any price movement they see. Since they cannot distinguish between price movements caused by Insiders and Outsiders, they occasionally trade with the Noise traders. Again, since there are many of them, on these occasions the market behaves like there is a large mass of Noise traders.

Proposition 2 shows that in a neighborhood of $I = 0$, $\frac{\partial p}{\partial S}$ is increasing in I, but does not tell us anything globally. We can numerically calculate these derivatives for a range of parameter values.

Table 1
$\frac{\partial p}{\partial S}, \sigma_1 = 0.1, \sigma_S = 0.1, \sigma_2 = 1$

Price sensitivity to noise $\frac{\partial p}{\partial S}$. The equilibrium sensitivity of the asset's price to noise shocks S depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

$I =$	$N = 0$	.01	.05	.1	.2
0	0	0.01	0.05	0.1	0.2
0.01	0	0.4999	0.2324	0.189	0.2398
0.05	0	0.189	0.4997	0.4417	0.3778
0.1	0	0.0972	0.3876	0.4995	0.4811
0.2	0	0.0489	0.2245	0.3777	0.4990

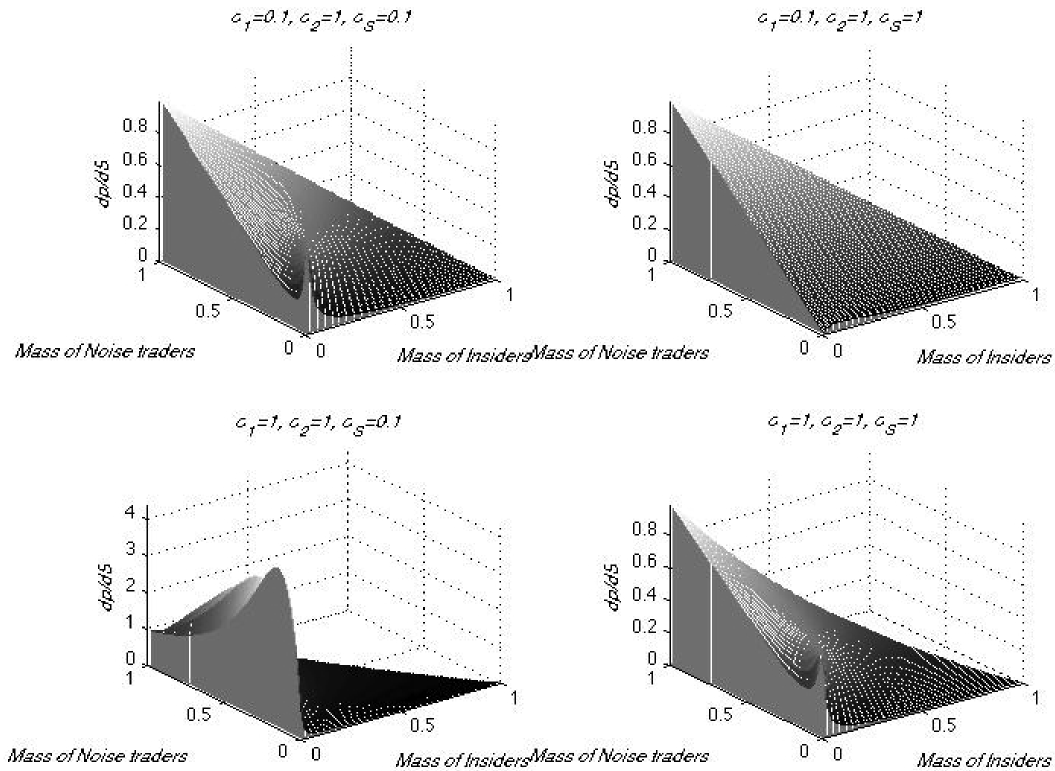

Fig. 5: Sensitivity of price to noise $\frac{\partial p}{\partial S}$. The equilibrium sensitivity of the asset's price to noise shocks S depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

The figures make clear that the effects described are strongest when the Noise traders are not very noisy. When the average quality of Insider information σ_1 is high compared to the average size of the sentiment shock σ_S, the odds that any price movement is due to noise trading is low, so it is optimal most of the time for the Outsider to trade with the price movement. In these situations, large sentiment shocks do not happen often, but even moderate shocks can become enormously magnified, even more so than in markets with only Noise traders. In these markets, the sensitivity of the price to the sentiment shock S can far exceed both N and 1, as shown in the lower left-hand corner of Fig. 5 and in Table 2. In particular, the figure shows that $\frac{\partial p}{\partial S}$ becomes large in markets with "quiet" Noise traders ($\sigma_S = 0.1$)

Table 2
$\frac{\partial p}{\partial S}, \sigma_1 = 1, \sigma_S = .1, \sigma_2 = 1$

Price sensitivity to noise $\frac{\partial p}{\partial S}$. The equilibrium sensitivity of the asset's price to noise shocks S depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

$I =$	$N = 0$	.01	.05	.1	.2
0	0	.01	0.05	.1	.2
0.01	0	0.4963	2.2908	3.763	4.2932
0.05	0	0.0995	0.486	0.9381	1.707
0.1	0	0.0497	0.2435	0.4726	0.8804
0.2	0	0.0249	0.1218	0.2367	0.4435

Table 3
$\frac{\partial p}{\partial S}, \sigma_1 = .1, \sigma_2 = 1, \sigma_S = 1$

Price sensitivity to noise $\frac{\partial p}{\partial S}$. The equilibrium sensitivity of the asset's price to noise shocks S depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

$I =$	$N = 0$	.01	.05	.1	.2
0	0	.01	0.05	.1	.2
0.01	0	0.0198	0.0519	0.1009	0.2004
0.05	0	0.0478	0.059	0.1042	0.2018
0.1	0	0.0544	0.0664	0.1079	0.2033
0.2	0	0.0413	0.0759	0.1133	0.2056

and well-informed Insiders ($\sigma_1 = 1$). This illustrates the central point of the paper: in these types of markets, it is rational for Outsiders to interpret any movements they see as based on information, and so trade with them. The mass of Outsider trades makes the effect $\frac{\partial p}{\partial S}$ very large.

Quantitatively, we want to focus on the magnitudes displayed in the lower left-hand panel of Fig. 5 and Table 2. Analytically, we have examined $\frac{\partial^2 p}{\partial S \partial I}$ and $\frac{\partial^2 p}{\partial S \partial N}$, but it is the level of $\frac{\partial p}{\partial S}$ that is of fundamental interest. Among the parameter values considered, the maximum value of 4.3641 for $\frac{\partial p}{\partial S}$ is achieved

Table 4
$\frac{\partial p}{\partial S}, \sigma_1 = 1, \sigma_2 = 1, \sigma_S = 1$
Price sensitivity to noise $\frac{\partial p}{\partial S}$. The equilibrium sensitivity of the asset's price to noise shocks S depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

$I =$	$N = 0$	.01	.05	.1	.2
0	0	.01	0.05	.1	.2
0.01	0	0.3988	0.3688	0.2672	0.2748
0.05	0	0.0986	0.3939	0.5	0.4706
0.1	0	0.0496	0.2305	0.3878	0.5
0.2	0	0.0249	0.1203	0.2256	0.375

with 17% Noise traders and 1% Insiders in the market, in the market with $\sigma_1 = 1, \sigma_S = .1$. This greatly exceeds the value of 1 that would obtain if *only* Noise traders participated in the market and 0.17 if there were no Insiders in the market. In the case with small masses of each type, the interaction of Noise traders and Insiders causes the Outsiders to occasionally chase noise aggressively, so that noise shocks are greatly amplified. Because σ_S is small, the market generally behaves well, but occasionally the price of the asset diverges sharply from fundamental value.

In this sense, the question is one of ex-ante or ex-post stability. Ex ante, the additional Insiders make the Price system more informative (shown in section 5.2) and more stable most of the time. Ex post and for specific realizations of S, the additional Insiders increase $\frac{\partial p}{\partial S}$ and so increase the sensitivity of the price to these shocks. This is a measure of ex-post instability.

It is tempting to make normative judgements about the effects of Insiders based on this destabilizing effect, but to do so would be premature. Adding Insiders does increase the effect of the Noise trader sentiment, increasing market volatility at time 1, but it also leads to fundamental information being factored into the price more effectively, leading to less volatility at time 2. Fig. 6 below shows the effect of the fundamental shock ν_1 on the period-1 price for different market configurations and parameter values. In all cases, more Insiders moves the market toward more fully pricing their fundamental information. In this respect, they are stabilizing the market.

Recall that the cases in which Insiders are destabilizing are those in which σ_S is small compared to σ_1 and the composition of the market lies in the Region of interest. These are exactly the markets where Noise traders are generally very quiet. Most of the time, Noise traders get only a small shock, Insider information gets factored into the price effectively, and the market behaves well. It is only on a rare occasion (like the spring of 2007, we argue) that the Noise traders get a moderate or big shock and the market behaves inefficiently because the rational Outsiders trade along with the noise.

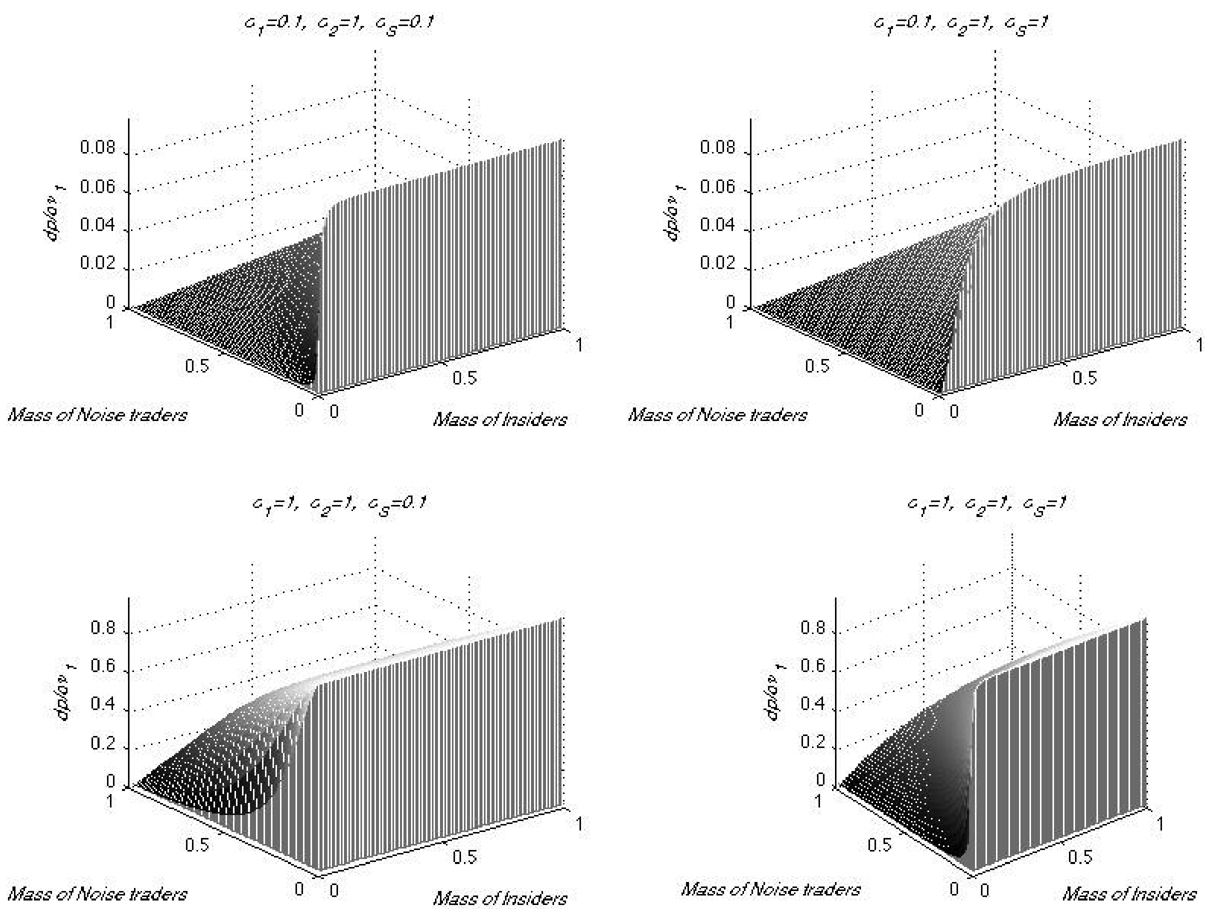

Fig. 6: Sensitivity of price to information: $\frac{\partial p}{\partial \nu_1}$. The equilibrium sensitivity of the asset's price to information shocks ν_1 depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

4.4. Demand covariance

We would like to think that most of the time, Outsiders successfully trade with the Insiders. The result of the previous section showed that when they fail to do so, they can fail rather dramatically. Here we show that, on average, they do indeed trade together. A reasonable way to measure whether Outsiders and Insiders trade together is the covariance of their demands. In Appendix B we prove the following proposition

Proposition 3. *The Outsiders, on average, trade with the Insiders. Specifically,* $Cov(D_I, D_O) \geq 0$.

This is extremely intuitive. If the Outsiders are rational, they must be doing their best to emulate the Insiders. If their demands did not positively covary, it would be profitable for the Outsiders to flip the slope of their demand curves.

Outsiders earn rational risk premia just for holding the asset, a term we do not focus on. Proposition 3 implies that they also make money, on average, by trading with the Insiders and against the Noise traders. Noise traders systematically lose money since they always trade against the information contained in ν_1. Their losses are mitigated only in those states when S and ν_2 have the same sign. Since the two shocks are uncorrelated, this occurs only half the time, and only rarely will be large enough to offset their losses due to trading against the Insiders.

How can Proposition 3 and Propositions 1, 2 be true at the same time?

Most of the time the Outsiders trade with the Insiders (this is Proposition 3). They do not, however, trade on the same side of the market 100% of the time. On rare occasions (for the parameter values we are interested in), the Noise traders get a modestly big shock. Because of the signal extraction at the heart of the model, the Outsiders believe that this is most likely an Insider shock, so trade with the Noise traders. This means that the effect of a moderate Noise trader shock is big (Propositions 1, 2), but only rarely is there a big enough Noise trader shock to cause the Outsiders to trade against the Insiders.

Proposition 2 is a statement about $\frac{\partial p}{\partial S}$ and how it changes as we vary the number of Insiders in the market. Now, we expect this derivative to be non-negative regardless of the composition of the market, because a slight increase in S shifts up the Noise trader's demand curve while leaving everyone else's unaffected.

Fundamentally, Propositions 3 and 1, 2 are about different things. Proposition 3 is for each fixed set of parameter values (specifically, the makeup of the market). Propositions 1 and 2 are answering questions about two simultaneous experiments: how much bigger would the price have been if S had been higher by dS? With that question answered, how much bigger would the answer to that question be if I were higher by dI?

5. Other measures of market stability and efficiency

5.1. Good variance, bad variance

Another metric to measure the impact Noise traders have on market efficiency is the variance of the equilibrium asset price. That variance can be written as

$$\sigma_p^2 = (\frac{\partial p}{\partial S})^2 \sigma_S^2 + (\frac{\partial p}{\partial \nu_1})^2. \tag{26}$$

This variance naturally splits into two pieces: variance caused by sentiment shocks, and variance caused by Insider information being factored into the price. The latter is "good variance," as it reduces volatility between times 1 and 2. The remaining "bad variance" can be looked at as the variance perceived by the Insider:

$$var(p|\nu_1) = (\frac{\partial p}{\partial S})^2 \sigma_S^2. \tag{27}$$

From this equation, it is clear that analyzing $var(p|\nu_1)$ is nearly equivalent to analyzing $\frac{\partial p}{\partial S}$. Holding σ_S constant and varying other parameters, increases in $\frac{\partial p}{\partial S}$ map one-to-one into increases in $var(p|\nu_1)$. As σ_S converges to zero, $\frac{\partial p}{\partial S}$ gets large, but that effect is offset by the decrease in σ_S. In Appendix C we show that the limit of $var(p|\nu_1)$ as σ_S converges to zero is zero, and that the convergence is asymptotically linear. This tempers the strength of some of our results, but leaves unchanged the conclusions about how the market behavior varies as we vary the market composition.

We would like to think that in a market composed of sophisticated investors, adding Insiders would be stabilizing and would decrease the variance of the price. Proposition 1 showed that $\frac{\partial p}{\partial S}$ can increase, so it comes as no surprise that the variance of price can also be increased by the addition of Insiders. In Appendix B we prove the following proposition

Proposition 4. *For sufficiently small N and I, changing a marginal Outsider into an Insider increases both the variance of the price and the "bad variance."*

The set of parameters for which the variance increases is identical to the set for which $\frac{\partial p}{\partial S}$ increases in Proposition 4.

By this metric as well, adding Insiders to a market is destabilizing. The interaction between Insiders and Outsiders in the presence of Noise traders causes the Noise trader shock to be integrated into the price more strongly, increasing the "bad variance." It also has the effect of increasing the sensitivity of the price to the Insider's information, increasing the "good variance." Naturally, this leads to the question of which effect is stronger. A natural way to compare the strength of these two effects is the Informativeness of the price system, which we consider next.

Fig. 7 shows the standard deviation of the price across different market compositions for various parameter values. In particular, it shows that with "quiet" Noise traders (σ_S small), the market can be more volatile with small numbers of Noise traders than with more of them. With more Noise traders, each Outsider's demand curve becomes downward sloping, so Outsiders partially offset the variance caused by Noise traders.

5.2. Informativeness of the price system

Grossman and Stiglitz (1976) define the "Informativeness of the price system" to be $(corr(p, \nu_1))^2$. This is a ratio of information-to-noise in prices and gives a measure of how well markets perform their function of reflecting information known to agents. The Informativeness in this case can be written as

$$\frac{(\frac{\partial p}{\partial \nu_1})^2}{\sigma_p^2} = \frac{1}{\frac{N^2}{I^2}\frac{\sigma_2^4\sigma_S^2}{(\sigma_1^2+\sigma_2^2)^2} + 1}. \tag{28}$$

From this expression two important propositions immediately follow:

Proposition 5. *Adding Insiders always weakly increases the Informativeness of the price system.*

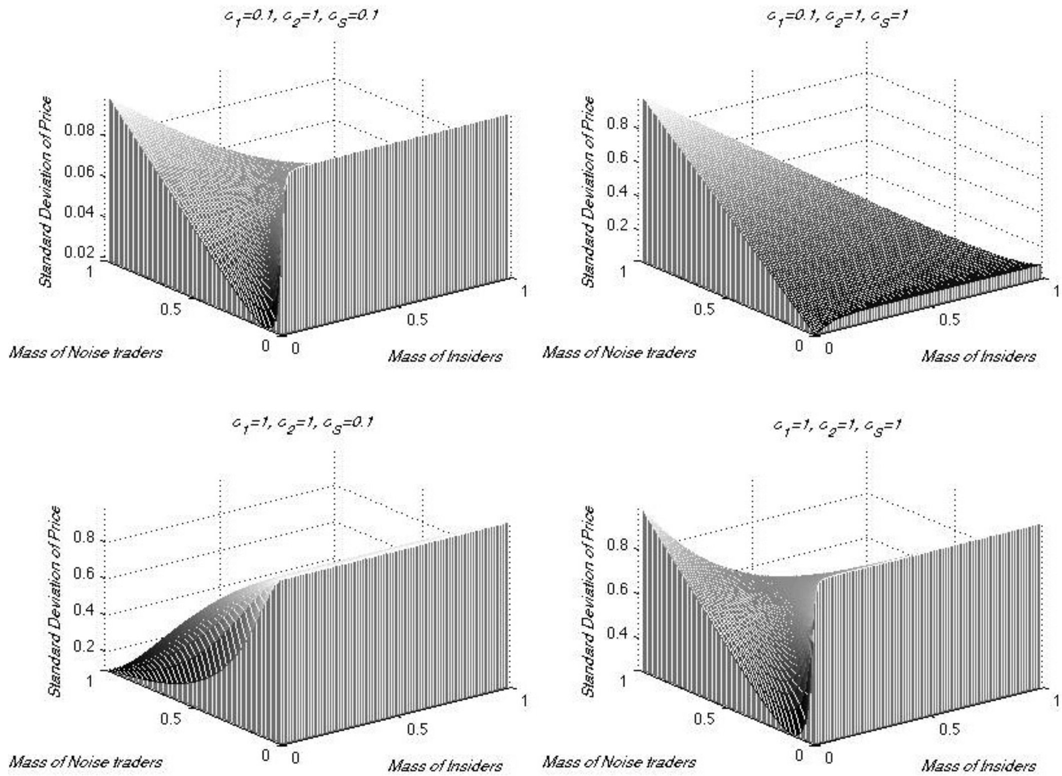

Fig. 7: Standard deviation of p. The equilibrium standard deviation of the asset's price in period two depends on the composition of the market and the relative standard deviations of the signals S, ν_1, ν_2. These are $\sigma_S, \sigma_1, \sigma_2$, respectively.

Proposition 6. *Adding Noise traders always decreases the Informativeness of the price system. This effect becomes unboundedly large as I and N approach zero.*

Any increase in the number of Insiders increases the Informativeness of the price system. This can be seen as the combination of two effects. First, chasing behavior by the Outsiders causes the "bad variance" to increase, which would tend to dampen the Informativeness of the price system. At the same time, this chasing behavior is applied to any information that the Insiders have. The Outsiders chase the Insiders, and the "good variance" increases. Proposition 5 says that the good variance increases by more than the bad variance.

Proposition 6 considers an alternative experiment of adding Noise traders (while removing Outsiders). It is no surprise that additional Noise traders decrease the Informativeness of the price, but it is by no means obvious that the effect can become unboundedly large as I goes to zero.

We have analyzed three ways of measuring the stability and efficiency of the market, with an eye towards seeing whether a small number of Noise traders can have an effect. The principal conclusion is that the presence of Noise traders can in fact have a large influence on the market equilibrium. Ex ante, small numbers of Noise traders do little to diminish the Informativeness of the price system, but can hugely increase the variance of the price in period 1. The result we focus more on is the surprising one: ex post, markets with a small number of Noise traders can have large sensitivities to the Noise trader shock, $\frac{\partial p}{\partial S}$. This, perhaps, can explain the evidence in Fig. 1.

5.3. Why $\frac{\partial p}{\partial S}$ is important

Given that there is at least one metric which cleanly identifies the efficiency of the market, why bother with any other metrics, in particular, $\frac{\partial p}{\partial S}$? The model is stylized and effectively static, but if we think of it as repeating itself, the time series behavior will be best described by the variance and Informativeness results. It is only when trying to understand specific market realizations that $\frac{\partial p}{\partial S}$ is important.

The ex-ante metrics show us that, on average, the Noise traders may have a fairly small effect in the Region of interest. It is no surprise that the price reacts to the Noise trader shock. What is surprising is that the sensitivity of the price to the Noise trader shock is not monotonically decreasing in the number of Insiders. In order to understand particular instances of sophisticated markets going awry, it is important to keep in mind that $\frac{\partial p}{\partial S}$ is liable to be big exactly in the markets in which we think Noise traders are quietest.

6. Conclusion

We presented a simple model in which rational but uninformed traders occasionally chase noise as if it were information, thereby amplifying sentiment shocks and moving price away from fundamental values. The model offers a potential explanation for the surprisingly low market price of financial risk in the spring of 2007.

We fill a gap in the theoretical literature by showing conditions under which Noise traders can have an impact on market equilibrium disproportionate to their size in the market. Explaining market outcomes by calling on large numbers of Noise traders or large sentiment shocks is not always plausible, but we show that neither of these is necessary in order for Noise traders to be relevant.

Our model is thus most suitable for modeling markets largely populated by sophisticated investors. It might help explain how sophisticated investors end up chasing noise in other situations of quiet before the storm, such as the period prior to the 1998 Russian crisis that bankrupted Long Term Capital Management. It is not good for shedding light on markets that might be dominated by noise traders, such as the Internet stocks.

A key feature of the model is the way in which sophisticated but uninformed investors learn from prices. Of course, such investors may entertain more complex models and use other public information, such as bond ratings, in forming their demands. This may lead to similar phenomena. If ratings agencies usually do a good job of assessing the riskiness of bond offerings, it may be rational for uninformed traders to use these ratings as a rule-of-thumb to assess underlying value. On those occasions when the ratings agencies are wrong, this will induce correlated mistakes among the mass of uninformed traders, which will overwhelm the price impact of any better-informed traders in the market. Only when the direct news about valuations reaches the uninformed investors would the market correct itself. In this example, uninformed traders end up rationally chasing noise thinking that it reflects information.

A. Derivation of conditional expectation and variance

We begin by deriving the demand curves directly from utility maximization. Let p be the price of the asset. The value is as above. Agent i begins with wealth W_i and chooses demand D_i to maximize

$$E_i[-e^{-\gamma(D_iV+(W_i-D_ip)r)}].$$

Maximizing this expression is equivalent to minimizing minus this expression, which is in turn equivalent to minimizing the log of that. Assuming for the moment that V is normally distributed *conditional on agent i's information set*, then we are trying to minimize

$$-\gamma E_i[(D_i V + (W_i - D_i p)r)] + \tfrac{\gamma^2}{2} D_i^2 \sigma_i^2(V),$$

where E_i denotes the expectation with respect to agent i's information set and $\sigma_i^2(V)$ denotes the variance of V conditional on agent i's information set. The first-order condition in D_i is

$$0 = -\gamma E_i[V] + \gamma pr + \gamma^2 D_i \sigma_i^2(V)$$

$$\Rightarrow D_i = \frac{E_i[V] - pr}{\gamma \sigma_i^2(V)}.$$

For the Insider, this becomes

$$D_i = \frac{\mu + \sigma_1 \nu_1 - pr}{\gamma \sigma_2^2}. \tag{29}$$

For the Outsider, this becomes

$$D_i = \frac{\mu + E[\sigma_1 \nu_1 | p] - pr}{\gamma \sigma_O^2}, \tag{30}$$

where $E[\sigma_1 \nu_1 | p]$ and σ_O^2 are endogenous. σ_O^2 is given by

$$\sigma_O^2 = Var(\sigma_1 \nu_1 | p) + \sigma_2^2.$$

Finally, the demand for the Noise traders is given by

$$D_i = \frac{\mu + S - pr}{\gamma \sigma_N^2}, \tag{31}$$

where σ_N^2 is the variance perceived by the Noise traders. Since the Noise traders do not observe a signal or use the price to update their information set, their perceived variance is the same as the ex-ante variance $\sigma_N^2 = (\sigma_1^2 + \sigma_2^2)$. With all this in hand, we can proceed to solve the model. Imposing market clearing gives

$$1 = N \tfrac{\mu + S - pr}{\gamma(\sigma_1^2 + \sigma_2^2)} + I \tfrac{\mu + \sigma_1 \nu_1 - pr}{\gamma \sigma_2^2} + O \tfrac{\mu + E[\sigma_1 \nu_1 | p] - pr}{\gamma \sigma_O^2}$$

$$\Rightarrow \gamma - \mu(\frac{N}{(\sigma_1^2+\sigma_2^2)} + \frac{I}{\sigma_2^2} + \frac{O}{\sigma_O^2}) + pr(\frac{N}{(\sigma_1^2+\sigma_2^2)} + \frac{I}{\sigma_2^2} + \frac{O}{\sigma_O^2}) - \frac{O}{\sigma_O^2}E[\sigma_1\nu_1|p] = \frac{N}{(\sigma_1^2+\sigma_2^2)}S + \frac{I}{\sigma_2^2}\sigma_1\nu_1. \tag{32}$$

This is Eq. (7) in the text. We can solve the signal extraction problem here to find the expectation of ν_1 given p. It is given by

$$E[\frac{I}{\sigma_2^2}\sigma_1\nu_1|p] = \frac{(\frac{I}{\sigma_2^2}\sigma_1)^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2} \times signal, \tag{33}$$

where the signal is the difference between the left-hand side of (7) and its unconditional expectation. That difference is (using the law of iterated expectations to eliminate the unconditional expectation of ν_1)

$$pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) - \tfrac{O}{\sigma_O^2}E[\sigma_1\nu_1|p] - E[pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) - \tfrac{O}{\sigma_O^2}E[\sigma_1\nu_1|p]]$$

$$= pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) - \tfrac{O}{\sigma_O^2}E[\sigma_1\nu_1|p] - E[pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2})].$$

We can find the unconditional expectation of p by taking expectations of (7):

$$\Rightarrow E[p] = \frac{\mu(\frac{N}{(\sigma_1^2+\sigma_2^2)} + \frac{I}{\sigma_2^2} + \frac{O}{\sigma_O^2}) - \gamma}{r(\frac{N}{(\sigma_1^2+\sigma_2^2)} + \frac{I}{\sigma_2^2} + \frac{O}{\sigma_O^2})}.$$

Plugging this in to the expression for the signal gives

$$pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) - \tfrac{O}{\sigma_O^2}E[\sigma_1\nu_1|p] - \mu(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) + \gamma.$$

So we can finally solve for the conditional expectation of ν_1:

$$\Rightarrow E[\sigma_1\nu_1|p] = \frac{\frac{I}{\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2 + \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}(pr(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) - \mu(\tfrac{N}{(\sigma_1^2+\sigma_2^2)} + \tfrac{I}{\sigma_2^2} + \tfrac{O}{\sigma_O^2}) + \gamma).$$

We can now solve for the price by plugging this in to the market clearing condition:

$\Rightarrow pr(\frac{O}{\gamma\sigma_O^2}+\frac{I}{\gamma\sigma_2^2}+\frac{N}{\gamma(\sigma_1^2+\sigma_2^2)})(1-\frac{\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}) = (\mu(\frac{N}{\gamma(\sigma_1^2+\sigma_2^2)}+\frac{I}{\gamma\sigma_2^2}+\frac{O}{\gamma\sigma_O^2})-1)(1-\frac{\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2})+\frac{N}{\gamma(\sigma_1^2+\sigma_2^2)}S+\frac{I}{\gamma\sigma_2^2}\sigma_1\nu_1.$

We cannot really rewrite this any more cleanly, but can define A and B by

$$A=\frac{O}{\gamma\sigma_O^2}+\frac{I}{\gamma\sigma_2^2}+\frac{N}{\gamma(\sigma_1^2+\sigma_2^2)}, \tag{34}$$

$$B=1-\frac{\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}, \tag{35}$$

so that we can solve for p as

$$p=r^{-1}(\mu-A^{-1})+\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}S+\frac{I}{ABr\gamma\sigma_2^2}\sigma_1\nu_1. \tag{36}$$

We are not yet done solving the signal extraction problem because we still need to solve for the conditional variance σ_O^2. We do that now. Recalling again Eq. (7):

$\gamma-\mu(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})+pr(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})-\frac{O}{\sigma_O^2}E[\sigma_1\nu_1|p]=\frac{N}{(\sigma_1^2+\sigma_2^2)}S+\frac{I}{\sigma_2^2}\sigma_1\nu_1.$

The Outsider observes the price and in equilibrium knows his own conditional expectation, so knows the left-hand side of this equation. Thus, he knows the right-hand side, so we can find the conditional variance of $\frac{I}{\sigma_2^2}\sigma_1\nu_1$ given the sum on the right-hand side.

$Var(\frac{I}{\sigma_2^2}\sigma_1\nu_1|p)=\frac{I^2\sigma_1^2}{\sigma_2^4}-(\frac{I^2\sigma_1^2}{\sigma_2^4})^2\frac{1}{\frac{I^2\sigma_1^2}{\sigma_2^4}+\frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)^2}}$

$$\Rightarrow Var(\sigma_1\nu_1|p) = \sigma_1^2 - \sigma_1^2 \frac{\frac{I^2\sigma_1^2}{\sigma_2^4}}{\frac{I^2\sigma_1^2}{\sigma_2^4} + \frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)^2}}$$

$$= \sigma_1^2 \frac{N^2\sigma_S^2\sigma_2^4}{I^2\sigma_1^2(\sigma_1^2+\sigma_2^2)^2+N^2\sigma_S^2\sigma_2^4}.$$

So we can calculate σ_O^2

$$\sigma_O^2 = \sigma_1^2 \frac{N^2\sigma_S^2\sigma_2^4}{I^2\sigma_1^2(\sigma_1^2+\sigma_2^2)^2+N^2\sigma_S^2\sigma_2^4} + \sigma_2^2. \tag{37}$$

This completely describes the equilibrium.

A.1. *Outsider demand curve slope*

Plugging the conditional mean and variance into the expression for the Outsider's demand curve gives

$$\frac{\mu+E[\sigma_1\nu_1|p]-pr}{\gamma\sigma_O^2} = \frac{\mu + \frac{\frac{I}{\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}(pr(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})-\mu(\frac{N}{(\sigma_1^2+\sigma_2^2)}+\frac{I}{\sigma_2^2}+\frac{O}{\sigma_O^2})+\gamma)-pr}{\gamma\sigma_O^2}$$

$$\Rightarrow \frac{dD_O}{dp} = \frac{r}{\gamma\sigma_O^2}\left(\frac{\frac{I}{\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}\left(\frac{N}{(\sigma_1^2+\sigma_2^2)} + \frac{I}{\sigma_2^2} + \frac{O}{\sigma_O^2}\right) - 1\right)$$

$$= \frac{r}{\gamma\sigma_O^2}\frac{1}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}\frac{N}{(\sigma_1^2+\sigma_2^2)}\left(\frac{I}{\sigma_2^2}\sigma_1^2 - \frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S^2\right)$$

$$= \frac{r}{\gamma}\frac{1}{\frac{\sigma_1^2}{\sigma_2^2}I(1-N)+\frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)}}\frac{N}{(\sigma_1^2+\sigma_2^2)}\left(\frac{I}{\sigma_2^2}\sigma_1^2 - \frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S^2\right).$$

This is the expression used in the text.

A.2. *Equilibrium conditional expectation*

The Outsider's conditional expectation of V in equilibrium is given by

$$E[V|p] = \mu + \frac{(1-B)\sigma_O^2}{O}\gamma\,(prA - \mu A + 1)$$

$= \mu + \frac{(1-B)\sigma_O^2}{O}\gamma\left((r^{-1}(\mu - A^{-1}) + \frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}S + \frac{I}{ABr\gamma\sigma_2^2}\sigma_1\nu_1)rA - \mu A + 1\right)$

$= \mu + \frac{(1-B)}{B}\frac{\sigma_O^2}{O}\left(\frac{N}{(\sigma_1^2+\sigma_2^2)}S + \frac{I}{\sigma_2^2}\sigma_1\nu_1\right).$

As usual, we have to evaluate this expression numerically, and we find that with the parameterizations considered in the paper, the coefficient on ν_1 can get as high as 1.9, while the slope on S can get as high as 5.8. When either of these slopes exceeds 1, for sufficiently large realizations of the shocks, the Outsiders may have expectations for the value of the asset which exceed those of either the Insiders or the Noise traders.

B. Proofs of propositions

B.1. $\frac{\partial p}{\partial S}$

B.1.1. $\frac{\partial^2 p}{\partial S \partial I}$

First, we start with the derivative of σ_O^2:

$\frac{\partial \sigma_O^2}{\partial I} = \frac{\partial}{\partial I}(\sigma_1^2 \frac{N^2\sigma_S^2\sigma_2^4}{I^2\sigma_1^2(\sigma_1^2+\sigma_2^2)^2+N^2\sigma_S^2\sigma_2^4} + \sigma_2^2)$

$= -2IN^2\sigma_1^4\sigma_S^2\sigma_2^4(\sigma_1^2+\sigma_2^2)^2(I^2\sigma_1^2(\sigma_1^2+\sigma_2^2)^2 + N^2\sigma_S^2\sigma_2^4)^{-2}.$

Note that this is zero at $I = 0$. Next, we look at the derivatives of A and B.

$\frac{\partial A}{\partial I} = \frac{\partial}{\partial I}(\frac{O}{\gamma\sigma_O^2} + \frac{I}{\gamma\sigma_2^2} + \frac{N}{\gamma(\sigma_1^2+\sigma_2^2)})$

$= -\frac{1}{\gamma\sigma_O^2} - \frac{O}{\gamma\sigma_O^4}\frac{\partial\sigma_O^2}{\partial I} + \frac{1}{\gamma\sigma_2^2}.$

At $I = 0$, this becomes

$= -\frac{1}{\gamma(\sigma_1^2+\sigma_2^2)} + \frac{1}{\gamma\sigma_2^2}.$

Moving on to B:

$\frac{\partial B}{\partial I} = \frac{\partial}{\partial I}(1 - \frac{\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2})$

$= -\frac{\partial}{\partial I}(\frac{\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2}{(\frac{I}{\sigma_2^2}\sigma_1)^2+(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2+\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2})$.

This is ugly to evaluate in general, but we can evaluate it at $I = 0$:

$= -((\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2 + \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2)^{-1}\frac{\partial}{\partial I}(\frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2) - \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2(\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2 + \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2)^{-2}\frac{\partial}{\partial I}(\frac{I}{\sigma_2^2}\sigma_1)^2 + (\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^2 + \frac{OI}{\sigma_O^2\sigma_2^2}\sigma_1^2)$

$= -(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^{-2}(\frac{O}{\sigma_O^2\sigma_2^2}\sigma_1^2)$

$= -(\frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S)^{-2}(\frac{O}{(\sigma_1^2+\sigma_2^2)\sigma_2^2}\sigma_1^2)$

$= -\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{N^2\sigma_2^2\sigma_S^2}$.

Finally, we can take a derivative of $\frac{\partial p}{\partial S}$ with respect to I:

$\frac{\partial^2 p}{\partial S \partial I}$

$= \frac{\partial}{\partial I}\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}$

$= \frac{N}{r\gamma(\sigma_1^2+\sigma_2^2)}\frac{\partial}{\partial I}\frac{1}{AB}$

$= -\frac{N}{r\gamma(\sigma_1^2+\sigma_2^2)}\frac{A\frac{\partial B}{\partial I}+B\frac{\partial A}{\partial I}}{(AB)^2}$.

Again, hard to evaluate, but at $I = 0$, this becomes

$= -\frac{N}{r\gamma(\sigma_1^2+\sigma_2^2)}\frac{-A\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{N^2\sigma_2^2\sigma_S^2} - B\frac{1}{\gamma(\sigma_1^2+\sigma_2^2)} + B\frac{1}{\gamma\sigma_2^2}}{(AB)^2}$.

When $I = 0$, A and B become:

$A = \frac{O}{\gamma(\sigma_1^2+\sigma_2^2)} + \frac{N}{\gamma(\sigma_1^2+\sigma_2^2)} = \frac{1}{\gamma(\sigma_1^2+\sigma_2^2)}$,

$B = 1$.

So, we can finally plug in to get

$$\frac{\partial^2 p}{\partial S \partial I} = -\frac{N}{r\gamma(\sigma_1^2+\sigma_2^2)} \frac{-A\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{N^2\sigma_2^2\sigma_S^2} - \frac{1}{\gamma(\sigma_1^2+\sigma_2^2)} + \frac{1}{\gamma\sigma_2^2}}{A^2}$$

$$= -\frac{N}{r\gamma(\sigma_1^2+\sigma_2^2)} \frac{-\frac{1}{\gamma(\sigma_1^2+\sigma_2^2)}\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{N^2\sigma_2^2\sigma_S^2} - \frac{1}{\gamma(\sigma_1^2+\sigma_2^2)} + \frac{1}{\gamma\sigma_2^2}}{(\frac{1}{\gamma(\sigma_1^2+\sigma_2^2)})^2}$$

$$= -\frac{N(\sigma_1^2+\sigma_2^2)}{r}(-\frac{O\sigma_1^2}{N^2\sigma_2^2\sigma_S^2} - \frac{1}{(\sigma_1^2+\sigma_2^2)} + \frac{1}{\sigma_2^2})$$

$$= \frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{rN\sigma_2^2\sigma_S^2} + \frac{N}{r} - \frac{N(\sigma_1^2+\sigma_2^2)}{r\sigma_2^2}$$

$$= \frac{1}{Nr}(\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{\sigma_2^2\sigma_S^2} + \frac{N^2\sigma_2^2}{\sigma_2^2} - \frac{N^2(\sigma_1^2+\sigma_2^2)}{\sigma_2^2})$$

$$= \frac{\sigma_1^2}{Nr\sigma_2^2}(\frac{O(\sigma_1^2+\sigma_2^2)}{\sigma_S^2} - N^2).$$

Proposition 2 can then be read directly off of this expression.

B.1.2. $\frac{\partial^2 p}{\partial S \partial N}$

We can do a similar analysis turning an Outsider into a Noise trader, starting from zero Noise traders. If $\frac{\partial^2 p}{\partial S \partial N}|_{N=0}$ is large and positive, this shows that markets with almost no noise need not behave almost like markets with no noise. In order, analyzing the derivatives of σ_O^2, A, and B at $N = 0$ gives

$$\frac{\partial \sigma_O^2}{\partial N}|_{N=0} = 0,$$

$$\frac{\partial A}{\partial N}|_{N=0} = -\frac{1}{\gamma\sigma_2^2} + \frac{1}{\gamma(\sigma_1^2+\sigma_2^2)} = -\frac{\sigma_1^2}{\gamma\sigma_2^2(\sigma_1^2+\sigma_2^2)},$$

$$\frac{\partial B}{\partial N}|_{N=0} = -\frac{-\frac{I\sigma_1^2}{\sigma_2^4}(\frac{I^2\sigma_1^2}{\sigma_2^4} + \frac{I(1-I)\sigma_1^2}{\sigma_2^4}) + \frac{OI\sigma_1^2}{\sigma_2^4}(\frac{I\sigma_1^2}{\sigma_2^4})}{(\frac{I^2\sigma_1^2}{\sigma_2^4} + \frac{I(1-I)\sigma_1^2}{\sigma_2^4})^2}$$

$$= -\frac{-\frac{I^2\sigma_1^4}{\sigma_2^8}+\frac{(1-I)I^2\sigma_1^4}{\sigma_2^8}}{(\frac{I^2\sigma_1^2}{\sigma_2^4}+\frac{I(1-I)\sigma_1^2}{\sigma_2^4})^2}$$

$$= -\frac{-I^2+(1-I)I^2}{(I^2+I(1-I))^2}$$

$$= -I.$$

So, we can solve for the desired comparative static:

$$\frac{\partial^2 p}{\partial S \partial N}|_{N=0} = \frac{1}{ABr\gamma(\sigma_1^2+\sigma_2^2)} - \frac{N}{A^2B^2r\gamma(\sigma_1^2+\sigma_2^2)}(A\frac{\partial B}{\partial N} + B\frac{\partial A}{\partial N})|_{N=0}$$

$$= \frac{1}{ABr\gamma(\sigma_1^2+\sigma_2^2)}|_{N=0}$$

$$= \frac{1}{\frac{1}{\gamma\sigma_2^2}(1-\frac{\frac{I(1-I)\sigma_1^2}{\sigma_2^4}}{\frac{(I^2+I(1-I))\sigma_1^2}{\sigma_2^4}})r\gamma(\sigma_1^2+\sigma_2^2)}$$

$$= \frac{\sigma_2^2}{Ir(\sigma_1^2+\sigma_2^2)}.$$

This can be arbitrarily big if I is close to zero. This shows Proposition 1.

B.2. *Variance of p*

$$p = r^{-1}(\mu - A^{-1}) + \frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}S + \frac{I}{ABr\gamma\sigma_2^2}\sigma_1\nu_1$$

$$\Rightarrow \sigma_p^2 = (\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)})^2\sigma_S^2 + (\frac{I}{ABr\gamma\sigma_2^2})^2\sigma_1^2.$$

As above, we consider changing dI Outsiders into Insiders.

$$\frac{\partial\sigma_p^2}{\partial I} = 2(\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)})\sigma_S^2\frac{\partial}{\partial I}(\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}) + 2(\frac{I}{ABr\gamma\sigma_2^2})\sigma_1^2\frac{\partial}{\partial I}(\frac{I}{ABr\gamma\sigma_2^2}).$$

We evaluate this at $I = 0$

$$\frac{\partial\sigma_p^2}{\partial I}|_{I=0} = 2(\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)})\sigma_S^2\frac{\partial}{\partial I}(\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)}).$$

The derivative here is the same as $\frac{\partial^2 p}{\partial S \partial I}$ from above when evaluated at $I = 0$. We then get

$$= 2(\frac{N}{ABr\gamma(\sigma_1^2+\sigma_2^2)})\sigma_S^2(\frac{O\sigma_1^2(\sigma_1^2+\sigma_2^2)}{rN\sigma_2^2\sigma_S^2} + \frac{N}{r} - \frac{N(\sigma_1^2+\sigma_2^2)}{r\sigma_2^2})$$

$$= \frac{\gamma(\sigma_1^2+\sigma_2^2)}{r^2\gamma}(\frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)} + \frac{O\sigma_1^2}{\sigma_2^2} - \frac{N^2\sigma_S^2}{\sigma_2^2})$$

$$= \frac{(\sigma_1^2+\sigma_2^2)}{r^2}(\frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)} + \frac{O\sigma_1^2}{\sigma_2^2} - \frac{N^2\sigma_S^2}{\sigma_2^2})$$

$$= \frac{\sigma_S^2\sigma_1^2}{r^2\sigma_2^2}(\frac{O(\sigma_1^2+\sigma_2^2)}{\sigma_S^2} - N^2).$$

We are primarily interested in cases when O is big and N is small. Again, the truth of the proposition can be read directly off of this last expression.

B.3. *Demand covariance*

$$Cov(D_I, D_O) = Cov(\frac{\mu+\sigma_1\nu_1-pr}{\gamma\sigma_2^2}, \frac{\mu+E[\sigma_1\nu_1|p]-pr}{\gamma\sigma_O^2})$$

$$= \frac{1}{\gamma^2\sigma_2^2\sigma_O^2}Cov(\sigma_1\nu_1 - pr, E[\sigma_1\nu_1|p] - pr).$$

Ignoring the constant term, we can write

$$Cov(\sigma_1\nu_1 - pr, E[\sigma_1\nu_1|p] - pr) = Cov(\sigma_1\nu_1 - pr, E[\sigma_1\nu_1 - pr|p]).$$

The second term in this covariance is the conditional expectation of the first term, that is, the function $F(p)$ that satisfies

$$E[\sigma_1\nu_1 - pr - F(p)] = 0,$$

and minimizes

$$E[(\sigma_1\nu_1 - pr - F(p))^2],$$

over all functions $F(p)$ which satisfy the first condition. We can write the second-moment criterion as

$E[(\sigma_1\nu_1 - pr)^2] + E[F(p)^2] - 2E[(\sigma_1\nu_1 - pr)F(p)]$

$= E[(\sigma_1\nu_1 - pr)^2] + E[F(p)^2] - 2Cov(\sigma_1\nu_1 - pr, F(p)) - 2E[\sigma_1\nu_1 - pr]E[F(p)].$

From the first criterion, $E[F(p)] = E[\sigma_1\nu_1 - pr]$, so this becomes

$= E[(\sigma_1\nu_1 - pr)^2] - 2E[\sigma_1\nu_1 - pr]^2 + E[F(p)^2] - 2Cov(\sigma_1\nu_1 - pr, F(p)).$

Suppose for the moment that the covariance we care about is negative. Then we are subtracting two of that covariance in this expression, or adding a positive term. Replacing $F(p)$ with $E[F(p)]$ will then decrease the $E[F(p)^2]$ term (by Jensen's Inequality) and turn the covariance to zero, thus decreasing the second moment we are trying to minimize. It follows that the covariance is non-negative, as desired. We have the desired proposition.

C. Tying $\frac{\partial p}{\partial S}$ to the slope of the Outsider demand curve

Write the Outsider demand curve as $D_O = mp + b$. The market clearing condition becomes

$1 = I\frac{\mu+\sigma_1\nu_1-pr}{\gamma\sigma_2^2} + N\frac{\mu+S-pr}{\gamma(\sigma_1^2+\sigma_2^2)} + O(mp+b).$

Taking a derivative in S gives

$0 = -I\frac{r}{\gamma\sigma_2^2}\frac{\partial p}{\partial S} + \frac{N}{\gamma(\sigma_1^2+\sigma_2^2)} - N\frac{r}{\gamma(\sigma_1^2+\sigma_2^2)}\frac{\partial p}{\partial S} + Om$

$$\Rightarrow \frac{\partial p}{\partial S} = (\frac{I}{\sigma_2^2}+\frac{N}{\sigma_1^2+\sigma_2^2})^{-1}\frac{N}{r(\sigma_1^2+\sigma_2^2)}+(\frac{I}{\sigma_2^2}+\frac{N}{\sigma_1^2+\sigma_2^2})^{-1}r^{-1}\gamma O\times Slope. \quad (38)$$

C.1. $\frac{\partial p}{\partial S}\sigma_S$

We have shown that $\frac{\partial p}{\partial S}$ gets large as σ_S gets small. We consider the product of these two terms

$\lim_{\sigma_S\to 0}\frac{\partial p}{\partial S}\sigma_S$

$$= \lim_{\sigma_S \to 0} (\frac{I}{\sigma_2^2} + \frac{N}{\sigma_1^2+\sigma_2^2})^{-1} \frac{N}{r(\sigma_1^2+\sigma_2^2)} \sigma_S + (\frac{I}{\sigma_2^2} + \frac{N}{\sigma_1^2+\sigma_2^2})^{-1} r^{-1} \gamma O \times OutsiderDemandCurveSlope \sigma_S$$

$$= \lim_{\sigma_S \to 0} (\frac{I}{\sigma_2^2} + \frac{N}{\sigma_1^2+\sigma_2^2})^{-1} r^{-1} \gamma O \sigma_S [\frac{r}{\gamma} \frac{1}{\frac{\sigma_1^2}{\sigma_2^2} I(1-N) + \frac{N^2\sigma_S^2}{(\sigma_1^2+\sigma_2^2)}} \frac{N}{(\sigma_1^2+\sigma_2^2)} (\frac{I}{\sigma_2^2}\sigma_1^2 - \frac{N}{(\sigma_1^2+\sigma_2^2)}\sigma_S^2)]$$

$$= 0.$$

Moreover, this convergence is asymptotically linear.

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